of zero entries in K. It is also possible to say that one stream segment may be so far from another that we would put a zero entry in the corresponding entry of K too. Here, we capture the fact that in different parts of the stream network, stream segments are flow related, not flow related, or both, by specifying which entries of K are zero or nonzero.

4. CONCLUSIONS

The underlying questions here are all concerned with why we want to allow (or not, as the case may be) correlation between specific locations, and how large we would like those correlations to be. In general, we like the variance-components model given by VHP, since it can easily handle nonstationarities in their Equation (10), and it can deal with flow-connected and flow-unconnected dependencies through mixtures. (It is easily seen that mixing on different types of models, tail-up, tail-down, and Euclidean, results in a covariance function that is equivalent to the variance-components model.) We have also shown how the spatial random effects model of Cressie and Johannesson (2008) can be adapted to modeling on stream networks, opening up possibilities for modeling nonstationary dependencies parsimoniously.

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Comment

Sujit K. SAHU

1. INTRODUCTION

The flow of water in streams and rivers poses a unique problem in defining the association between underwater monitored quantities at any two sites. The usual methods of using Matérn covariance functions for the random quantities measured above water do not work, because the flow of the water and movement of creatures such as fish in both upstream and downstream directions must be allowed to influence the association appropriately. This very original and impressive article develops and illustrates some new moving average models for stream networks. New covariance models are presented based on the stream distance rather than the Euclidean distance.

We begin our discussion by raising some questions on the models developed. We conclude by considering possible extensions of the variance component models to other inferential settings.

2. VARIANCE COMPONENT MODELS

The authors cleverly construct a variance component model corresponding to eq. (10),

$$Y(\mathbf{s}) = \mathbf{x}(\mathbf{s})^T \boldsymbol{\beta} + z_u(\mathbf{s}) + z_d(\mathbf{s}) + \epsilon(\mathbf{s}), \tag{1}$$

where $z_u(\mathbf{s})$ and $z_d(\mathbf{s})$ are underlying independent tail-up and tail-down processes with previously defined covariance functions $C_u(\cdot)$ and $C_d(\cdot)$, and $\mathbf{x}(\mathbf{s})$ are location-specific covariate values. Below eq. (10), the authors also discuss the possibility of adding another component accounting for unmeasured covariates, $z_o(\mathbf{s})$ say, (where the suffix o represents omnidirectional) that could be related due to underlying bedrock characteristics. This additional component can serve many other purposes as well; for example, we may want to model characteristics of connected streams, rivers, and lakes at the same time. Wider segments of the rivers and the lakes connecting the upstream and downstream areas will require the use of the term $z_o(\mathbf{s})$, because the random observation at any location can depend on that from any other location, not just those upstream or downstream. This gives rise to the general model

$$Y(\mathbf{s}) = \mathbf{x}(\mathbf{s})^T \boldsymbol{\beta} + z_u(\mathbf{s}) + z_d(\mathbf{s}) + z_o(\mathbf{s}) + \epsilon(\mathbf{s}). \tag{2}$$

But the additional term, $z_o(\mathbf{s})$, may make one or both of $z_u(\mathbf{s})$ and $z_d(\mathbf{s})$ nonsignificant, because the omnidirectional term may capture all of the dependence. The data alone may not be rich enough to separate out the directional dependencies. This parallels a very common problem in spatial statistics on assessing anisotropy using directional variograms. For example, Banerjee, Carlin, and Gelfand (2004, sec. 2.3.2) remarked that "directional variograms from data generated under a simple isotropic model will routinely exhibit differences of magnitude seen in Figure 2.9(a)." One possible solution to this problem might be

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Journal of the American Statistical Association
March 2010, Vol. 105, No. 489, Applications and Case Studies
DOI: 10.1198/jasa.2010.ap09621

to introduce weights for various components, analogous to the discussion in Section 2.2.1. Suitable prior covariance structure for the $z_o(\mathbf{s})$, $z_u(\mathbf{s})$, and $z_d(\mathbf{s})$ processes also may help identify them.

A further drawback of the foregoing formulation is the a priori assumption of independence of the tail-up, tail-down, and omnidirectional components. It is not hard to imagine applications in which these cannot be assumed independent and there can be confounding effects between the three components; for example, the same fish (creature) can travel both upstream and downstream and "horizontally" as well. In such cases a multivariate specification must be provided. There are well-known problems of multivariate spatial specifications, and either a separable model or a linear model of co-regionalization can be specified (see, e.g., Gelfand et al. 2004 and references therein, including Ver Hoef and Barry 1998).

3. EXTENSION TO THE SPACE–TIME DATA

The authors discuss the possibility of extending the model to space–time data. Indeed, the model representation in eq. (2) can easily do that,

$$Y(\mathbf{s},t) = \mathbf{x}(\mathbf{s},t)^T \boldsymbol{\beta}_t + z_u(\mathbf{s},t) + z_d(\mathbf{s},t) + z_o(\mathbf{s},t) + \epsilon(\mathbf{s},t), \quad (3)$$

where the spatial processes at a particular time point are extended to spatiotemporal processes indexed by time point t ($t = 1, 2, \ldots$). A careful choice of the dynamic processes is necessary for model description, identification, estimation, and prediction. The covariate process $\mathbf{x}(\mathbf{s},t)$ may depend on time and may need to be modeled as well. (See, e.g., Huerta, Sanso, and Stroud 2004 and Sahu, Gelfand, and Holland 2007, where meteorological variables such as temperatures are modeled simultaneously with ozone concentration levels.) The process $\boldsymbol{\beta}_t$ can be assumed to be $\boldsymbol{\beta}_t = \rho\boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t$, where $\boldsymbol{\eta}_t$ are independent Gaussian random variables. The parameter ρ can be assumed to be 0 for independence of $\boldsymbol{\beta}_t$'s, 1 for random walk, and some nonzero value in the interval $(-1, 1)$ corresponding to autoregressive processes. The tail-up $[z_u(\mathbf{s},t)]$, tail-down $[z_d(\mathbf{s},t)]$, and omnidirectional $[z_o(\mathbf{s},t)]$ processes can be assumed to be independent over time as a simple starting model. Complex, multivariate space–time interaction can be built up by joint modelling of the three processes. (Chap. 8 in Banerjee, Carlin, and Gelfand 2004 is an excellent starting point for this sort of modeling.) The pure error process, $\epsilon(\mathbf{s},t)$, is usually assumed to be independent in space and time, providing the so-called "nugget effect."

4. EXTENSION TO THE GENERALIZED LINEAR MODELS

The first-stage Gaussian models described so far are not appropriate for discrete data. It also is very common to observe presence–absence data for species or chemicals in a stream network. In those cases, the Gaussian distribution assumption for the $Y(\mathbf{s},t)$ must be replaced by an appropriate member of the exponential family of distributions, and the model specification (3) is now written as

$$g\big(E(Y(\mathbf{s},t))\big) = \mathbf{x}(\mathbf{s},t)^T \boldsymbol{\beta}_t + z_u(\mathbf{s},t) + z_d(\mathbf{s},t) + z_o(\mathbf{s},t) \quad (4)$$

for a suitable link function $g(\cdot)$. Process assumptions made on the second and subsequent stages can remain the same except for the nugget effect $\epsilon(\mathbf{s},t)$, which will no longer be there, although there are computational reasons for keeping the term.

The likelihood function for these variance components models will not be tractable for estimation purposes. The Bayesian computation methods based on Markov chain Monte Carlo (MCMC) techniques can be used as an alternative. But the Bayesian methods will require specification of prior distributions for all of the parameters and hyperparameters. Once a MCMC method has been successfully implemented, it is a relatively straightforward task to perform predictions using posterior predictive distributions.

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Rejoinder

Jay M. VER HOEF and Erin E. PETERSON

We thank the editor David Banks for inviting this interesting discussion, and the discussants Noel Cressie, David O'Donnell, and Sujit Sahu for their very insightful comments. Following Cressie and O'Donnell's discussion, we refer to our article as VHP throughout.

1. STREAMS AS DIRECTED GRAPHS

Cressie and O'Donnell bring up the interesting connection between stream networks and directed graphs. They make their point clearly, and the analogous model complexity and computational difficulties that arise from chain graphs and mixed

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Journal of the American Statistical Association
March 2010, Vol. 105, No. 489, Applications and Case Studies
DOI: 10.1198/jasa.2010.ap09781